Original Article

Monte Carlo simulation risk analysis for underground mining projects

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Abstract

This study presents the application of the Monte Carlo method as a procedure of risk analysis in economic evaluation, covering, through the evaluation methodology presented, the stages of conceptual and pre-feasibility projects. In addition to the literature review, the methodology was applied to a cash flow prepared for an underground gold mine project in Brazil, exploited through the long hole mining method. Thus, deterministic evaluation methodologies were compared with the probabilistic output provided by the Monte Carlo method. The risk analysis made it possible to evaluate the impact of variations in the economic model's input data: ore content, tonnage, ore price, OPEX (Operational Expenditure), CAPEX (Capital Expenditure), commissioning time, and mining recovery. In the example studied, the simulated scenarios indicated a 98.6% chance of the NPV (Net Present Value) being greater than zero and a mean NPV of $ M 261.186 versus $ M 287.513 from the deterministic cash flow evaluation. All these analyses are essential to evaluate the risks of failure, investments and economic and financial viability of a mining enterprise, being essential for planning and decision making in similar studies.
Keywords: Risk analysis; Stochastic economic evaluation; Underground mining projects; Feasibility study.

Análise de risco por simulação de Monte Carlo para projetos de mineração subterrânea

Resumo

Esse estudo apresenta a aplicação do método de Monte Carlo como método de análise de riscos em avaliação econômica, abrangendo, através da metodologia de avaliação apresentada, as etapas de projetos conceituais e de pré-viabilidade. Para tanto, além da verificação em bibliografia, se aplicou a metodologia a um fluxo de caixa, elaborado para um projeto de mina subterrânea. Assim, foram comparadas metodologias de avaliação determinísticas com a saída probabilística fornecida pelo método de Monte Carlo. A análise de risco viabilizou avaliar o impacto de variações de dados de entrada do modelo econômico: teor, reserva, preço do minério, OPEX, CAPEX, tempo de comissionamento e recuperação da lavra. No exemplo estudado, a simulação de cenários indicou cerca de 98,6% de chances de o NPV ser maior do que zero e NPV médio de $ M 261,186 contra $ M 287,513 da avalição determinística do fluxo de caixa. Todas essas análises são essenciais para avaliar os riscos de insucesso, os investimentos e a viabilidade econômico-financeiros de um empreendimento de mineração, sendo fundamental para o planejamento e tomada de decisão em estudos similares.
Palavras-chave: Análise de risco; Avaliação econômica; Projetos de minas subterrâneas; Estudo de viabilidade.

1 Introduction

The study of technical and economic feasibility, which converges to the structuring and evaluation of cash flow, is considered the basis for the implementation of projects of any kind. In the mining industry, one frequently works with large scales of operation and, therefore, million-dollar projects, where even the prospective studies for extra data mean high expenditures due to the cost with drilling and laboratory tests. Thus, preliminary estimates and the consideration of variability become of great relevance. In underground mining, as well as in open pit mines, there is an important number of variables to consider, as mineral deposit geometries, grade, production rate, uncertainty, and investments. While these factors are considered relevant in the conceptual and pre-feasibility design stages, as well as

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in the sensitivity analysis, rarely they are seen as part of risk economic evaluations, mainly because risk analysis is not a default procedure until today in the mining industry.

In a technical and economic evaluation of a mine venture there are at least three types of studies correlated to the level of detail during the process [1]. The first one is the Preliminary Economical Assessment (PEA) where the main target is to establish potential economic viability of mineral resources. A second step is the Prefeasibility Study (PFS) where the studies are sufficient to demonstrate the economic viability and establish mineral reserves. The third one is the Feasibility Study (FS) where a detailed study will be built to be used as the basis for a production decision. At the PEA level, a cost estimation accuracy between 30 and 50% is acceptable, and the so called quick estimates can be structured by the way of cost modelling.

For quick estimates, the mining sector uses estimation by similarity of values and parameters with projects already executed or with cost estimates executed in detail (detail projects), as presented by Carriconde [2]. Motta and Calôba [3] present how the cost of investment in a given project (CAPEX - capital expenditure and OPEX - operational expenditure) is proportional to the cost of investment in a project of the same nature, but with a different production scale. Estimators can be used to determine the magnitude of CAPEX at an early stage, commonly referred as the six-tenths rule [4]. Other works use more robust models based on parametric equations to cost estimates, such as O'Hara [5], O'Hara and Suboleski [6], Camm and Stebbins [7], and Araújo et al. [8].

Blank and Randazzo [9], point out that the use of cash flow, with estimated operating costs (OPEX) and revenues, is a useful resource for evaluating the potential of projects to be developed, remembering that projected cash flows in preliminary stages are a simplification of future flows. Sá [10] affirms that, for various sectors, the study of cash flow and the elements that act in its formation are fundamental to the evaluation of investments and projects. Moreover, the author also emphasizes that cash flow is a useful tool for the management of corporate liquidity. Classic project analysis models include cash flow analysis by the traditional methods of NPV (Net Present Value), IRR (Internal Rate of Return), PAYBACK and sensitivity analysis. However, Barboza [11] states that few studies address risk conditions or how the project behaves when faced with uncertainties regarding the input data and parameters of the cash flow model. These variations are described by probabilistic distributions, which, coupled with the cash flow model, result in a distribution for the outcome of the output variable.

Thereby, this paper proposes to evaluate the application of Monte Carlo simulation in the cash flow study of an underground mining project. It was considered the methodologies for evaluating projects in the conceptual phases, i.e., with cost inference - which are naturally expected to present a certain degree of intrinsic variability.

1.1 Risks analysis

According to Revuelta and Jimeno [12], cost estimating is fundamental to engineering projects. According to D'Arrigo [13], for a mining project cost estimation, the higher the percentage of engineering completed, the lower the variability in the estimated cost. Table 1 shows the characteristic accuracy for different project phases, according to the Australian Mining Industry Cost Estimating Manual [1]. Observing the table, we naturally see great variability in the initial stages of projects, and consequently in their evaluation of economic viability. It is essential to measure and take into account the risks, which will have a direct bearing on the variability of these project stages.

Monte Carlo Risk Analysis allows you to obtain the probability distribution of a dependent variable through a successive simulations process (Figure 1). In each simulation, a particular set of values of the independent variables, drawn at random, is used to obtain a value of the dependent variable. Based on this function, probability analyses are performed on the variable. According to Silva [15], there is no exact number of interactions to be performed in the method, and the best result is associated with the greatest number of possible iterations. However, a minimum number should be observed to have a representative result, which is a function of the error (ϵ), in percentage, attributed to the project. In other words, a larger number of simulations reduces the error and allows one to obtain approximate distributions by repeating the analysis of the same model. Regarding the statistical distribution models to be considered for use in cash flow simulations, their formulation and best use situations, Mun [16] is quote as a consistent reference.

In mining projects, Risk Assessment is highly advisable, ranging from analyses by qualitative methods, assessing generic risks to the project, as presented by Sturk et al. [17] and Domínguez et al. [18]. The applicability of probabilistic methods such as the Monte Carlo method was possible due to the computational processing power and the availability of robust databases with parameters of interest, which has been directing risk analyses towards the probabilistic field. Increasingly other areas, such as engineering, have been using these tools and evaluating scenario analyses, previously restricted by deterministic methodologies. The works of Vargas et al. [19], Zhang and Huang [20], and

Table 1. Precision of cost estimates in different project phases

Project Phase	Precision
Conceptual Project	65 – 70%
Pre-Feasibility	75 – 80%
Feasibility	85 – 90%
Engineering Project	90 – 95%

Source: Lanz and Noakes [1].

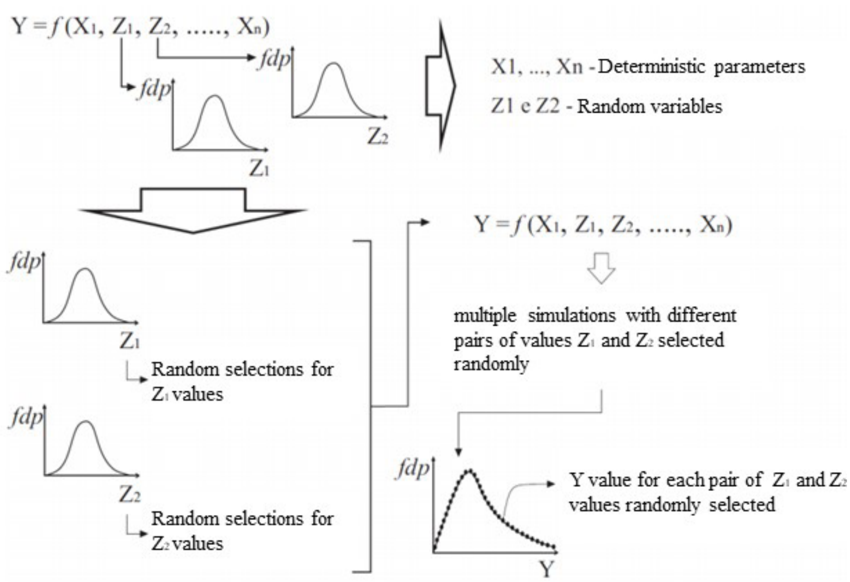


Figure 1. Illustration of the Monte Carlo Method systematization. Source: Charbel [14].

Charbel [14], focused on design applications, can be used as examples. Currently, the tool is widely adapted for the economic evaluation of projects, where it can be adapted to cash flows to provide probabilistic results, i.e., probabilistic distributions of economic indicators such as NPV, IRR, PAYBACK, among others. The works of Cardin et al. [21], Wei et al. [22], Petter [23], Souza [24], and Assis [25] are examples of application of this tool in mining cash flows.

As a risk analysis systematization for project cash flow evaluation, Cardin et al. [21] propose the use of 4 stages: Stage 1, determining the deterministic scenario, i.e., the cash flow and uncertainty variables; Stage 2, identifying the sensitivity of projects to possible alternatives and variations; Stage 3, creating a catalog of possible variations in the project, defining intervals and variations that those variables of interest can assume, such as the price of commodities; Stage 4, assessing the project value (NPV) against the risk, via scenario simulations.

1.2 Economic assessment in underground mining

As early as 1874, J. H. Collins [26], in his book *Principles of metal mining*, highlighted the importance of underground mining and the need to properly design the mining process and its structures. New technologies have led to higher productivity and lower costs, which makes it possible to exploit more deposits, making underground mining more present in the context of modern mining.

Regarding to project evaluation, underground mining presents a greater dynamic of options than open pit mining, with more alternatives of methods and technologies, which propose a greater adherence to a given situation, generally related to the ore body features and planned production. This greater dynamic is reflected in the techno-economic modeling, where we see a wide range of parameters for modeling, which represents a major influence on CAPEX and OPEX, such as entries, mechanization, development of accesses, ground control, grade variation, production and recovery rates.

Tatiya [27] presents a comparative summary of underground mining methods in terms of technical characteristics and application environments. This comparison may well be a starting point for cost ideation; however, one must consider that for many environments and types of deposits, more than one mining method may present themselves as adequate, as well as more than one method may be applied, being even common for some cases the use of different associated methods.

2 Methodology

The present study followed the division of stages proposed by Cardin et al. [21]. For the initial stage (stage 1) - structuring a cash flow - the methodology presented by Araújo et al. [8] for underground mines was used to estimate costs. This methodology allows, above all, to impute variation/uncertainty regarding production and geometric characteristics of the mine, varying CAPEX and OPEX. As a case study, it was considered an underground evaluation using the Long Hole method. The method is a variation of the sub-level stoping, where panels occur, interleaved in levels, separated by pillars: sill pillar, longitudinal pillar and/or rib pillar. The ore body must have a dip greater than 50° and a tabular shape with a minimum thickness of 3 meters and extensive in length [28]. Long holes are drilled between the two excavations and loaded with explosives. Currently among the most widely used methods in underground mining [8].

Table 2 presents the project data considered; it should be noted that the data are compatible with those of real projects publicly available for consultation, a source used by Araújo et al. [8] to formulate their equations. Equation 1, proposed by Long [29], refers to the estimated annual production for underground mines, used as the initial production rate input for the cash flow. Equations 2 and 3 correspond to the calculation of CAPEX and OPEX values by the models for Long Hole Mining by Araújo et al. [8]. The equations are function of total daily production in tons per day (tpd), tonnage in tons, and average stope thickness in meters.

$$Production(tpd) = 0.297 * Tonnage^{0.562} \quad (1)$$

$$CAPEX(US\$) = Tonnage * 13,892.0 * \left[prod(tpd) * stope(m)\right]^{-0.622} \quad (2)$$

$$OPEX(US\$/ton) = 161.3 * \left[prod(tpd) * stope(m)\right]^{-0.096} \quad (3)$$

Figure 2 presents the model adopted in the study where the variables and Cash Flow, in a simplified version, are fed by the parameters and their respective probability distributions. It is noteworthy that CAPEX and OPEX

Table 2. Inputs and their respective variations and distributions considered for the simulation

INPUT	DATA	DISTRIBUTION	OBSERVATIONS
Tonnage	Maximum: 12,000,000.0 ton Deterministic: 10,000,000.0 ton Minimum: 8,000,000.0 ton	Uniform (-20% +20%)	author's consideration
Stope	Mean: 5.0 m σ^2: 0.5 - truncated	Normal	by De Araújo et al. [8]
Index CAPEX	Maximum: 1.25 Deterministic: 1 Minimum: 0.85	Triangular	Index that multiplies the Capex considering the precision [29]
Index OPEX	Maximum: 1.25 Deterministic: 1 Minimum: 0.85	Triangular	Index that multiplies the Opex considering the precision [29]
ORE	Mean: 5 ppm σ^2: 0.25	LogNormal	author's consideration, distribution format based on that observed in the evaluation of gold deposits
Mining Recovery	Maximum: 85% Deterministic: 80% Minimum: 60%	Triangular	based on the expectation of recovering the method
Price	Mean: 56.7 US$/g σ^2: 5.0	LogNormal	price for last two years and variation shown
Taxes	Fixed: 21%	-	author's consideration

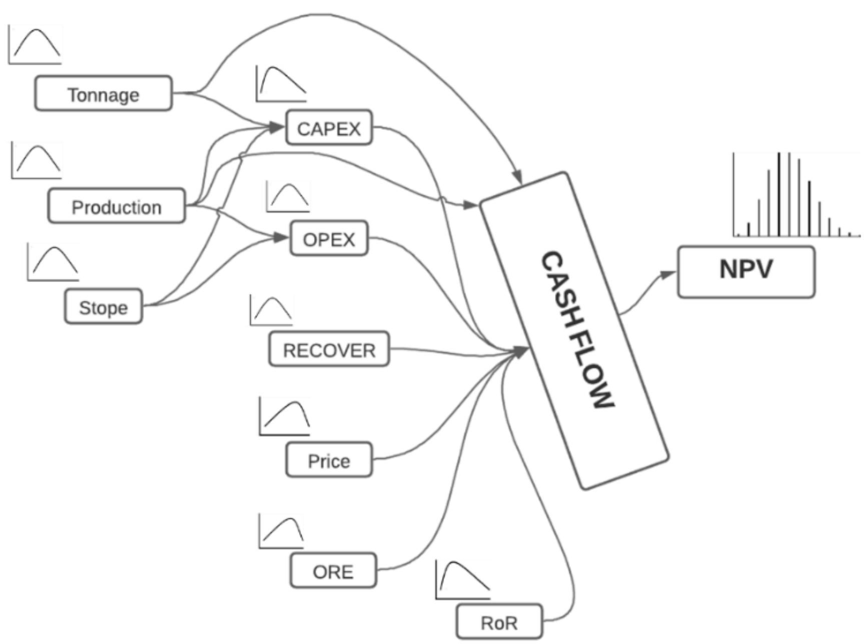


Figure 2. Structure of the Model adopted.

(Equations 1-3) are functions of Tonnage, Production and Stope, having been considered an uncertainty for their value, opting for a sequential variability. For Taxes, which compile fees and taxes of various natures, by considering that these tend to remain more stable, this input was considered fixed.

For the analysis, other assumptions were considered, such as:

A. Underground Gold Mine in Brazil.
B. Long Hole Mining Method with holes > 5 meters.
C. 3-year commissioning period and for commencement of payment of calculated CAPEX.
D. Homogeneous inflation.
E. Besides the variability of CAPEX and OPEX inputs, a variability intrinsic to them was also considered. An Index multiplying the estimates was used to represent the expected precision range as indicated for preliminary studies [30].
F. It was considered a rate of return (RoR) of 7.74% per year, estimated by the CAPM (Capital Asset Pricing Model) [31] methodology as a function of a risk-free rate of return (RF) of 2.93% per year and a risk premium of 4.81% per year. The risk premium considers a Beta coefficient of 0.6, an expected market return (ERm) of 6% and a country risk of 2.97%. As per Equation 4.

$$RoR = RF + \beta\left(ERm - RF\right) + COUNTRY\ RISK \tag{4}$$

In the cash flow evaluation, the OPEX (US$) indicated in each period corresponds to the unit OPEX (US$ per ton) multiplied by the annual production. The PROFIT corresponds to the multiplication of the inputs production (annual), ORE, Mining Recover and PRICE. It's the gross profit related to the contained metal.

For the TAXES, a simplification of the tax burden to a percentage of 21% was considered, corresponding to the employment and profit taxes. In the Cash Flow this parameter multiplies the net profit (difference between profit and OPEX). In the second stage (stage 2), a sensitivity analysis was performed to validate the sensitivity of the cash flow to the inputs defined for the risk analysis, using Tornado analysis.

Figure 3 presents the Tornado Graph, which shows the sensitivity of NPV to variations in parameters. Based on this, it is possible to reevaluate which parameters would be of more interest in relation to the assertiveness of their expected value, that is, which would denote a greater degree of knowledge and/or, in fact, be considered in a Risk Analysis. It is possible to see in the application of the example that the values of Price, Recovery and Ore content have a great impact on the NPV of the project. This allows us to evaluate whether these inputs should be prioritized in additional studies or in their variability measurement. For example, the variations in inputs such as the stope size have greater weight (considering equal limits of variation) than OPEX and the reserve in the result of the NPV.

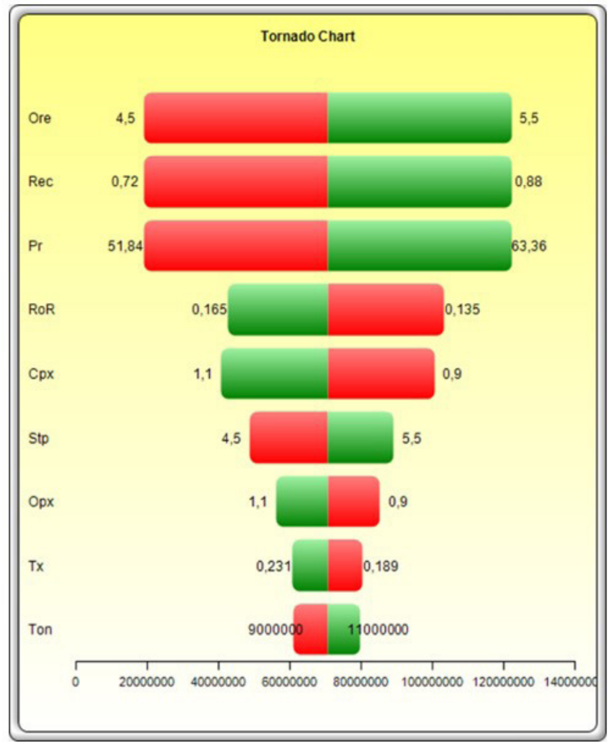


Figure 3. Tornado Sensitivity Analysis.

In the third stage (stage 3), the variabilities of the cash flow inputs were defined. Regarding to the scenarios evaluated, synthetic probabilistic distributions were considered in the input of the cash flow. The limits of the distributions considered the reports of underground mines by Long Hole method - database structured by Araújo et al. [7]. The distribution models, according to Table 2, were chosen according to the practice of mining projects.

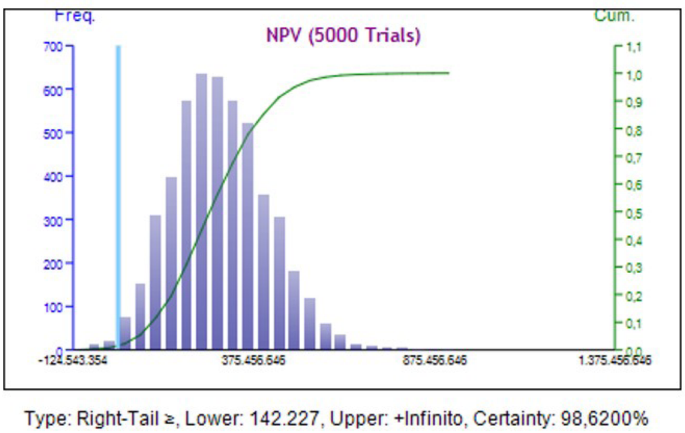
In the last stage (stage 4), after running the simulation with varying design parameters, a statistical and critical analysis of the results was performed. For the automation of the cash flow processes, an Excel spreadsheet was used, as well as the Risk Simulator add-in for sensitivity analysis and Monte Carlo simulation, running 5000 simulations. Considering that a high number of simulations represents a considerable reduction in error associated with different rounds of simulations [15].

3 Results and discussion

Table 3 presents the mine's structured cash flow. It can be seen that the deterministic NPV is positive, making the venture viable. However, it is emphasized that robust evaluations look not only at this, but at other indicators. In the deterministic case, the evaluated project showed an NPV of over $ 287.513 million, representing a profit - investment ratio of 74% and an IRR of 19%. Thus, on deterministic parameters, it can be defined as an economically viable project.

Table 3. Deterministic Cash Flow

Year	1	2	3	4	...	15
Reserve (t)	1,00E+07	1,00E+07	1,00E+07	9,18E+06	1,02E+06	2,03E+05
prod (t)	0	0	0	81,640,645.08	81,640,645.08	20,312,258.99
CAPEX ($)	155,261,190.47	116,445,892.85	116,445,892.85	0	0	0
OPEX ($)	-	-	-	53,136,268.83	53,136,268.83	13,220,346.96
Profit ($)	-	-	-	188,100,046.27	188,100,046.27	46,799,444.72
Taxes ($)	-	-	-	28,342,393.26	28,342,393.26	7,051,610.53
NET Profit ($)	155,261,190.47	116,445,892.85	116,445,892.85	106,621,384.18	106,621,384.18	26,527,487.23
NPV ($)	**287,513,387.96**					
ROE	**19%**					



Number of Trials	5000
Mean	261,186,664.40
Median	253,308,184.09
Standard Deviation	133,075,008.59
Variance	1.77E+16
Coefficient of Variation	0.5095
Maximum	914,938,852.54
Minimum	-152,291,339.07
Range	1,067,230,191.61
Skewness	0.3511
Kurtosis	0.3349
25% Percentile	168,492,489.85
75% Percentile	346,928,255.07
Percentage Error Precision at 95% Confidence	1.41%

Figure 4. NPV distribution probability and its parameters by Monte Carlo Simulation.

According to the result of the Risk Analysis (Figure 4), we have the frequency distribution of the simulated NPV values, emphasizing that this is a product generated from the distributions of the cash flow input values. We can evaluate, from this result, in how many scenarios we reach values ≥ the expected NPV, and thus ponder what are the chances of success of the project. Another possible evaluation consists on the verification of the scenarios in that the venture presented NPV above the minimum expected.

A major advantage of using the Risk Analysis is the contribution to the portfolio study, providing a consistent quantitative parameter to complement the traditional indicators (NPV, IRR, and PAYBACK). By assessing the statistics of different projects evaluated by risk analysis, we can compare their statistical results and especially their coefficient of variation. In the example, the simulated scenarios presented a standard deviation of $ M 133.075 and a mean of $ M 261.186 for the NPV, representing a coefficient of variation of 0.5095. Note that the average of the simulated values was above the deterministically calculated value and that in about 1.37% of the simulated scenarios the NPV was less than zero. In 59.6% of the simulated scenarios the NPV was below the deterministically calculated value. Note that the behavior of the IRR is analogous to that of the NPV, for purposes of assessing its variability.

4 Conclusion

The Risk Analysis in the methodology used proved to be a relevant valuable tool and of easy application to conceptual and pre-feasibility studies for underground mining and mining in general. By evaluating the impact of uncertainty as varying parameters (reserve, production, and depth) related to CAPEX and OPEX estimates, we see a realistic consideration of risk, natural to underground mining. One considers that there is a lack of studies on the actual behavior of inputs (distribution model). Applying the equations of Araújo et al. [7] in the methodology, used in the cash flow of this work, one can verify the viability of applying Risk Analysis in Economic Evaluations based on other cost estimation models, such as those of O'Hara [5], O'Hara and Suboleski [6] and Camm and Stebbins [7]. This would allow for consideration of variations in other initial parameters (inputs) such as the depth of the ore body, the number of workers, and others.

In the example, although the cash flow explored may be considered simplistic, it adequately demonstrates the applicability of risk analysis and its feasibility for underground mining project evaluations. Note that the same tool can be used on cash flows with a larger number of variables. Using other models for inference of design parameters and even cash flows, other underground mining related parameters can be evaluated, such as rock mass quality and mechanization grade. It should be noted that the relationship between excavation parameters, ground control and feasibility, and mining costs are topics of broad interest, considering that trends in sustainability point toward underground versus open pit mining.

This tool could be aggregated to technical reports of projects listed on stock exchanges such as those of the NI 43-101 standard. Normally limited to sensitivity assessments for economic reports of projects.

As risk considerations, it is recommended a careful evaluation of projects, in the case of rejecting a certain project for eventually presenting a low percentage of positive scenarios. If we evaluate the risks considered in the simulations, we should observe the tendency to over penalize the inputs. In the CAPEX and OPEX evaluation, for example, the intrinsic variability of these was considered as an example, as well as the variability of the parameters used for their estimation in Equations 1, 2 and 3; a fact that should be properly evaluated and considered in the conclusions to be taken from the simulations' results. Another factor that should be properly observed and evaluated is the usual discount rates of cash flow, which tend to take into consideration risk-free rates, i.e., they already consider in their formulation natural risks of the sector being evaluated. Thus, one needs to be clearly aware of whether one is considering risks over risks in the model - making a conservative assessment - or whether one is simplifying the model by extracting overlapping risks.

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